

82. *CPT* Invariance Tests in Neutral Kaon Decay

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CPT theorem is based on three assumptions: quantum field theory, locality, and Lorentz invariance, and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in $K^0 - \bar{K}^0$ system, described by the equation

$$i \frac{d}{dt} \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} = [M - i\Gamma/2] \begin{bmatrix} K^0 \\ \bar{K}^0 \end{bmatrix} ,$$

where M and Γ are hermitian matrices (see PDG review [1], references [2,3], and KLOE paper [5] for notations and previous literature), allows a very accurate test of *CPT* symmetry; indeed since *CPT* requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, the mass and width eigenstates, $K_{S,L}$, have a *CPT*-violating piece, δ , in addition to the usual *CPT*-conserving parameter ϵ :

$$K_{S,L} = \frac{1}{\sqrt{2(1+|\epsilon_{S,L}|^2)}} \left[(1 + \epsilon_{S,L}) K^0 \pm (1 - \epsilon_{S,L}) \bar{K}^0 \right]$$

$$\epsilon_{S,L} = \frac{-i\Im(M_{12}) - \frac{1}{2}\Im(\Gamma_{12}) \mp \frac{1}{2} \left[M_{11} - M_{22} - \frac{i}{2} (\Gamma_{11} - \Gamma_{22}) \right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2}$$

$$\equiv \epsilon \pm \delta. \tag{82.1}$$

Using the phase convention $\Im(\Gamma_{12}) = 0$, we determine the phase of ϵ to be $\varphi_{SW} \equiv \arctan \frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Imposing unitarity to an arbitrary combination of K^0 and \bar{K}^0 wave functions, we obtain the Bell-Steinberger relation [4] connecting *CP* and *CPT* violation in the mass matrix to *CP* and *CPT* violation in the decay; in fact, neglecting $\mathcal{O}(\epsilon)$ corrections to the coefficient of the *CPT*-violating parameter, δ , we can write [5]

$$\left[\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i \tan \phi_{SW} \right] \left[\frac{\Re(\epsilon)}{1 + |\epsilon|^2} - i\Im(\delta) \right] =$$

$$\frac{1}{\Gamma_S - \Gamma_L} \sum_f A_L(f) A_S^*(f), \tag{82.2}$$

where $A_{L,S}(f) \equiv A(K_{L,S} \rightarrow f)$. We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Eq. (82.2); in fact, defining for the hadronic modes

$$\alpha_i \equiv \frac{1}{\Gamma_S} \langle \mathcal{A}_L(i) \mathcal{A}_S^*(i) \rangle = \eta_i \mathcal{B}(K_S \rightarrow i),$$

$$i = \pi^0 \pi^0, \pi^+ \pi^-(\gamma), 3\pi^0, \pi^0 \pi^+ \pi^-(\gamma), \tag{82.3}$$

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the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations (the analysis described in Ref. 5 has been updated by using the recent measurements of K_L branching ratios from KTeV [6,7], NA48 [8,9], and the results described in the *CP* violation in K_L decays minireview, and the recent KLOE result [10])

$$\begin{aligned}
\alpha_{\pi^+\pi^-} &= ((1.112 \pm 0.010) + i(1.061 \pm 0.010)) \times 10^{-3} , \\
\alpha_{\pi^0\pi^0} &= ((0.493 \pm 0.005) + i(0.471 \pm 0.005)) \times 10^{-3} , \\
\alpha_{\pi^+\pi^-\pi^0} &= ((0 \pm 2) + i(0 \pm 2)) \times 10^{-6} , \\
|\alpha_{\pi^0\pi^0\pi^0}| &< 1.5 \times 10^{-6} \quad \text{at 95\% CL} .
\end{aligned} \tag{82.4}$$

The semileptonic contribution to the right-handed side of Eq. (82.2) requires the determination of several observables: we define [2,3]

$$\begin{aligned}
\mathcal{A}(K^0 \rightarrow \pi^- l^+ \nu) &= \mathcal{A}_0(1 - y) , \\
\mathcal{A}(K^0 \rightarrow \pi^+ l^- \nu) &= \mathcal{A}_0^*(1 + y^*)(x_+ - x_-)^* , \\
\mathcal{A}(\bar{K}^0 \rightarrow \pi^+ l^- \nu) &= \mathcal{A}_0^*(1 + y^*) , \\
\mathcal{A}(\bar{K}^0 \rightarrow \pi^- l^+ \nu) &= \mathcal{A}_0(1 - y)(x_+ + x_-) ,
\end{aligned} \tag{82.5}$$

where x_+ (x_-) describes the violation of the $\Delta S = \Delta Q$ rule in *CPT*-conserving (violating) decay amplitudes, and y parametrizes *CPT* violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\bar{K}^0)$ beams, CPLEAR has measured $\Im(x_+)$, $\Re(x_-)$, $\Im(\delta)$, and $\Re(\delta)$ [11]. These determinations have been improved in Ref. 5 by including the information $A_S - A_L = 4[\Re(\delta) + \Re(x_-)]$, where $A_{L,S}$ are the K_L and K_S semileptonic charge asymmetries, respectively, from the PDG [12] and KLOE [13]. Here we are also including the *T*-violating asymmetry measurement from CPLEAR [14].

Table 82.1: Values, errors, and correlation coefficients for $\Re(\delta)$, $\Im(\delta)$, $\Re(x_-)$, $\Im(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [5] and CPLEAR [14].

	value	Correlations coefficients			
$\Re(\delta)$	$(3.0 \pm 2.3) \times 10^{-4}$	1			
$\Im(\delta)$	$(-0.66 \pm 0.65) \times 10^{-2}$	-0.21	1		
$\Re(x_-)$	$(-0.30 \pm 0.21) \times 10^{-2}$	-0.21	-0.60	1	
$\Im(x_+)$	$(0.02 \pm 0.22) \times 10^{-2}$	-0.38	-0.14	0.47	1
$A_S + A_L$	$(-0.40 \pm 0.83) \times 10^{-2}$	-0.10	-0.63	0.99	0.43 1

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The value $A_S + A_L$ in Table 82.1 can be directly included in the semileptonic contributions to the Bell Steinberger relations in Eq. (82.2)

$$\begin{aligned} & \sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu) \mathcal{A}_S^*(\pi\ell\nu) \rangle \\ &= 2\Gamma(K_L \rightarrow \pi\ell\nu) (\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta))) \\ &= 2\Gamma(K_L \rightarrow \pi\ell\nu) ((A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta))) . \end{aligned} \quad (82.6)$$

Defining

$$\alpha_{\pi\ell\nu} \equiv \frac{1}{\Gamma_S} \sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu) \mathcal{A}_S^*(\pi\ell\nu) \rangle + 2i \frac{\tau_{K_S}}{\tau_{K_L}} \mathcal{B}(K_L \rightarrow \pi\ell\nu) \Im(\delta) , \quad (82.7)$$

we find:

$$\alpha_{\pi\ell\nu} = ((-0.2 \pm 0.5) + i(0.1 \pm 0.5)) \times 10^{-5} .$$

Inserting the values of the α parameters into Eq. (82.2), we find

$$\begin{aligned} \Re(\epsilon) &= (161.1 \pm 0.5) \times 10^{-5} , \\ \Im(\delta) &= (-0.7 \pm 1.4) \times 10^{-5} . \end{aligned} \quad (82.8)$$

The complete information on Eq. (82.8) is given in Table 82.2.

Table 82.2: Summary of results: values, errors, and correlation coefficients for $\Re(\epsilon)$, $\Im(\delta)$, $\Re(\delta)$, and $\Re(x_-)$.

	value	Correlations coefficients			
$\Re(\epsilon)$	$(161.1 \pm 0.5) \times 10^{-5}$	+ 1			
$\Im(\delta)$	$(-0.7 \pm 1.4) \times 10^{-5}$	+ 0.09	1		
$\Re(\delta)$	$(2.4 \pm 2.3) \times 10^{-4}$	+ 0.08	-0.12	1	
$\Re(x_-)$	$(-4.1 \pm 1.7) \times 10^{-3}$	+ 0.14	0.22	-0.43	1

Now the agreement with *CPT* conservation, $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$, is at 18% C.L.

The allowed region in the $\Re(\epsilon) - \Im(\delta)$ plane at 68% CL and 95% C.L. is shown in the top panel of Fig. 82.1.

The process giving the largest contribution to the size of the allowed region is $K_L \rightarrow \pi^+ \pi^-$, through the uncertainty on ϕ_{+-} .

The limits on $\Im(\delta)$ and $\Re(\delta)$ can be used to constrain the $K^0 - \bar{K}^0$ mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\bar{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\bar{K}^0})}{\Gamma_S - \Gamma_L} \cos \phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)] .$$

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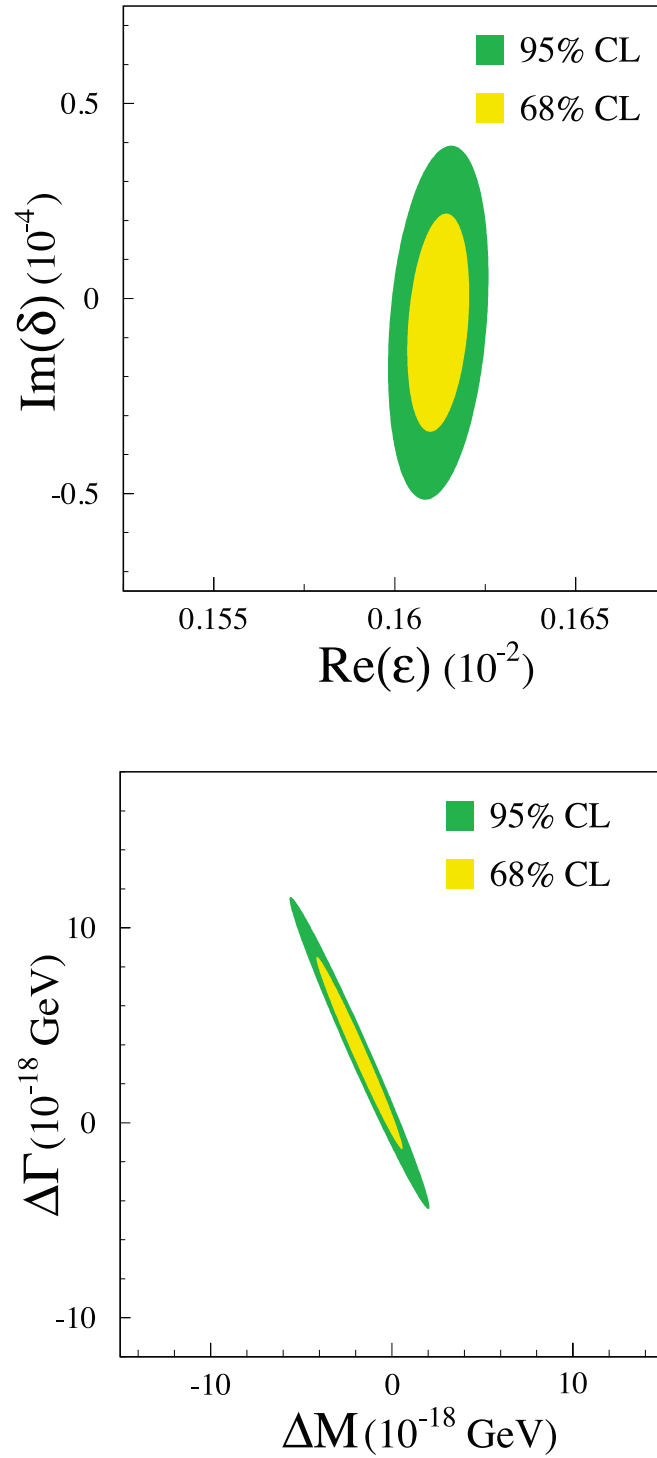


Figure 82.1: Top: allowed region at 68% and 95% C.L. in the $\Re(\epsilon)$, $\Im(\delta)$ plane. Bottom: allowed region at 68% and 95% C.L. in the ΔM , $\Delta\Gamma$ plane.

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The allowed region in the $\Delta M = (m_{K^0} - m_{\bar{K}^0}), \Delta\Gamma = (\Gamma_{K^0} - \Gamma_{\bar{K}^0})$ plane is shown in the bottom panel of Fig. 82.1. As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. 12) and in the limit $\Gamma_{K^0} - \Gamma_{\bar{K}^0} = 0$ we obtain

$$-4.0 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\bar{K}^0} < 4.0 \times 10^{-19} \text{ GeV} \quad \text{at } 95 \% \text{ C.L.}$$

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